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A SMALL COSMOLOGICAL CONSTANT FROM A LARGE EXTRA DIMENSION

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ABSTRACT

We propose a new approach to the Cosmological Constant Problem which makes essential use of an extra dimension. A model is presented in which the Standard Model vacuum energy "warps" the higher-dimensional spacetime while preserving $4D$ flatness. We argue that the strong curvature region of our solutions may effectively cut off the size of the extra dimension, thereby giving rise to macroscopic $4D$ gravity without a cosmological constant. In our model, the higher-dimensional gravity dynamics is treated classically with carefully chosen couplings. Our treatment of the Standard Model is however fully quantum field-theoretic, and the $4D$ flatness of our solutions is robust against Standard Model quantum loops and changes to Standard Model couplings.

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The extremely small value of the cosmological constant poses the most severe naturalness problem afflicting fundamental physics (see Ref. [1] for a review). The problem stems from the fact that in General Relativity, *all* forms of energy necessarily act as gravitational sources which curve spacetime. Generically, the energy density of the vacuum as represented by the cosmological constant, would yield an unacceptably high curvature. An old but still exciting idea for ameliorating this situation is to assume that we live in a fundamentally higher-dimensional spacetime, which is indeed greatly curved by vacuum energy [2]. However, it is now possible that the curvature spills into the extra dimensions, which can be larger than the fundamental Planck length [3, 4, 5, 6, 7]. This idea has never been realized in any way which does not require the same level of fine-tuning as in four dimensions, although the way the fine-tuning shows up does change in interesting ways [5, 6, 8, 9, 10, 11]. A very interesting recent approach to the problem has been explored in [12, 13, 14].

In this paper, we report on some limited progress in this direction. Working within a five-dimensional "brane universe" effective field theory [15] in which the Standard Model (SM) is confined to a 3-brane, we will show that one can carefully choose the higher-dimensional gravitational dynamics so that the SM vacuum energy is effectively converted into a current which is carried off the brane into the "bulk". The gravitational back-reaction of this current warps the higher-dimensional spacetime, but in a manner consistent with four-dimensional Poincare invariance of the vacuum solution and inconsistent with four-dimensional de-Sitter or Anti-de-Sitter symmetry. However, our vacuum solution necessarily contains a singular region parallel to the SM brane, signaling high sensitivity to short-distance gravity and the breakdown of effective field theory in that region. Nevertheless we will argue that the singular region may effectively cut off the higher dimensional spacetime as in [16, 17] (see also [18]), thereby rendering the gravitational dynamics macroscopically four-dimensional, with vanishing effective cosmological constant.

In this paper we will explicitly consider only classical (higher-dimensional) gravity, but with quantum SM effects included. The gravitational couplings must be chosen very precisely for the mechanism to work, although having chosen them the mechanism is completely stable under SM radiative corrections and changes in SM couplings. To be precise about the size of effects neglected by the truncation to classical gravity, it is useful to distinguish the contributions to the cosmological constant which do and do not involve quantum gravity, whose fundamental scale is M_*. In an effective field theory framework valid up to an energy $\Lambda < M_*$, we can expand the cosmological constant as

$$\lambda = \mathcal{O}(\Lambda^4) + \mathcal{O}(\frac{\Lambda^6}{M_*^2}) + ... \ . \tag{1}$$

The first term obtains a contribution from the SM vacuum energy, treated fully quantum field theoretically, but is insensitive to quantum gravity corrections. The subsequent terms are sensitive to quantum gravity effects, described in Feynman diagrams by virtual graviton exchanges suppressed by powers of $1/M_*^2$. Note that the formally larger first term

is under the best theoretical control, while the subleading terms are shrouded to some extent by the mysteries of quantum gravity. Our truncation limits us to understanding the screening of the $\mathcal{O}(\Lambda^4)$ SM vacuum energy, deferring the problem of $\mathcal{O}(\frac{\Lambda^6}{M_*^2})$ effects. This does not mean that these subleading effects are phenomenologically negligible since the smallest we can take Λ is our experimental cutoff of $\mathcal{O}(\mathrm{TeV})$, so that even $\frac{\Lambda^6}{M_*^2}$ is much larger than the observed cosmological constant. Nevertheless we consider it a significant advance to show how the effects of the SM vacuum energy alone can be nullified.

Our set-up is as follows. We consider a five-dimensional spacetime, or "bulk", where the fifth dimension is a half-line with coordinate y, while the usual four dimensions have coordinates x^μ. Orbifold boundary conditions as in [19, 5] will be used to realize the half-line in terms of a full real line with the identification of y with $-y$. The five-dimensional degrees of freedom will therefore be taken to be symmetric about $y = 0$. We will consider the boundary of the half-line, $y = 0$, to be the location of an "end-of-the-world" 3-brane, to which the SM quantum field theory (or some extension thereof) is confined. The SM degrees of freedom interact with classical five-dimensional bulk gravity as well as a classical five-dimensional scalar field ϕ. Similar models have been studied in different contexts in [19, 20, 10]. In the bulk, the scalar field will only be coupled minimally to bulk gravity, and on the brane it will be conformally coupled to the SM. Similar couplings have been employed in $4D$ attempts to solve the cosmological constant problem [21, 1]. We will carefully choose the conformal coupling constant such that for a given value of the vacuum energy on the brane there exists a flat solution. However, as we will show, once this choice is made, the ensuing flat solution remains unaffected by quantum loops on the brane, showing that the vanishing of the $4D$ cosmological constant is both radiatively stable and insensitive to the values of the SM parameters. Up to two derivatives, the action of our model is

$$S = \int d^4x dy \sqrt{g_5}\Big(\frac{R}{2\kappa_5^2} - \frac{3}{2}(\nabla\phi)^2\Big) - \int d^4x \sqrt{-\det(g_4(0)e^{\kappa_5\phi(0)})}\mathcal{L}_{SM}(H, g_{\mu\nu}(0)e^{\kappa_5\phi(0)}) \,. \quad (2)$$

The constant κ_5^2 is related to the $5D$ Planck scale M_* by $\kappa_5^2 = M_*^{-3}$. Here M, N run over the $5D$ coordinates, while μ, ν run over the $4D$ brane coordinates. Note the special normalization of the bulk scalar ϕ, or equivalently the special coupling to the brane for the canonically normalized scalar field. The SM Lagrangian $\mathcal{L}_{SM}$ is localized to the brane at $y = 0$. It depends on the SM fields, $H(x)$ and any necessary ultraviolet regulators, all minimally coupled to the Weyl-rescaled induced metric $g_{\mu\nu}(y = 0)e^{\kappa_5\phi(y=0)}$. Note that we have set the bulk cosmological constant and the operators $f(\kappa_5\phi)R$ equal to zero. With the Weyl transformation $\bar{g}_{MN} = g_{MN}\exp(\kappa_5\phi)$ we can cast the action (2) into the "string frame", where it takes the form

$$S = \int d^4x dy \sqrt{\bar{g}_5} e^{-3\kappa_5\phi/2} \frac{\bar{R}}{2\kappa_5^2} - \int d^4x \sqrt{\bar{g}_4(0)}\mathcal{L}_{SM}(H, \bar{g}_{\mu\nu}(0)) \,. \quad (3)$$

It is now clear from this action that our choice of the conformal coupling is completely unaffected by quantum loops on the brane. The brane action, (including ultraviolet

regulators), does not even contain the scalar field ϕ, so $4D$ general covariance guarantees that the SM renormalization will not change the form of the actions (3) and (2). For simplicity, we will use the canonical action (2) in what follows.

To incorporate the effects of SM quantum loops we can integrate out the SM degrees of freedom, and work in terms of the full 1PI effective action. Hence the brane action in (2) is replaced by the effective action,

$$S_{brane} \to \Gamma^{SM}_{eff}(H, g_{\mu\nu}(0) e^{\kappa_5 \phi(0)}) \,. \tag{4}$$

The equations of motion are now straightforward to obtain. Varying the action (2) and using (4) we find

$$\begin{aligned} R^{MN} - \frac{1}{2} g^{MN} R &= 3\kappa_5^2 \Big(\nabla^M \phi \nabla^N \phi - \frac{1}{2} g^{MN} (\nabla\phi)^2 \Big) + \frac{2\kappa_5^2}{\sqrt{g_5}} \frac{\delta \Gamma^{SM}_{eff}}{\delta g_{\mu\nu}} \delta^M{}_\mu \delta^N{}_\nu \delta(y) \,, && (5) \\ 3\Box_5 \phi &= -\frac{1}{\sqrt{g_4}} \frac{\delta \Gamma^{SM}_{eff}}{\delta \phi} \delta(y) \,, && (6) \\ \frac{\delta \Gamma^{SM}_{eff}}{\delta H} &= 0 \,. && (7) \end{aligned}$$

Equation (7) encapsulates the full SM quantum field theory in the curved background.

We now seek solutions with $4D$ Poincare symmetry. With this symmetry the SM effective action is given by just the effective potential,

$$\Gamma^{SM}_{eff} = -\int d^4x \sqrt{g_4} V_{eff}(H) e^{2\kappa_5 \phi} \,, \tag{8}$$

and hence the SM equations of motion take the simple form

$$\frac{\partial V_{eff}}{\partial H} = 0 \,. \tag{9}$$

We will denote an extremal value of the potential by $V_{extremal}$. Further, the structure of the $5D$ metric is restricted by the Poincare symmetry to the form

$$ds^2 = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \,, \tag{10}$$

where $a(y)$ is the warp factor. The equations of motion (5)-(6) then take a particularly simple form. Defining the shifted scalar field $\tilde\phi = \phi + \frac{1}{2\kappa_5} \ln(\frac{V_{extremal}}{M_*^4})$, the field equations in the bulk become

$$\frac{a'^2}{a^2} = \frac{\kappa_5^2 \tilde\phi'^2}{4} \,, \qquad \tilde\phi'' + 4\frac{a'}{a} \tilde\phi' = 0 \,, \qquad \frac{a''}{a} = -\frac{3\kappa_5^2 \tilde\phi'^2}{4} \,, \tag{11}$$

while the δ-function sources and the $y \to -y$ symmetry imply the matching conditions for the first derivatives

$$a'(0) = -\frac{M_*}{6} e^{2\kappa_5 \tilde\phi(0)} a(0) \,, \qquad \tilde\phi'(0) = \frac{M_*^{5/2}}{3} e^{2\kappa_5 \tilde\phi(0)} \,. \tag{12}$$

Here the prime refers to the derivative with respect to y. Note that with the definition of the variable $\tilde{\phi}$, the gravitational field equations are completely independent of the extremum value of the SM effective potential $V_{extremal}$. This will be crucial for ensuring the success of our mechanism.

It is straightforward to solve the system (11)-(12). The explicit solutions with Poincare symmetry are

$$\begin{aligned} ds_5^2 &= (1-\frac{2M_*}{3}e^{2\kappa_5\tilde{\phi}_0}|y|)^{1/2}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \\ \tilde{\phi} &= \tilde{\phi}_0 - \frac{1}{2\kappa_5}\ln\Big(1-\frac{2M_*}{3}e^{2\kappa_5\tilde{\phi}_0}|y|\Big)\,, \\ \phi &= \phi_0 - \frac{1}{2\kappa_5}\ln\Big(1-\frac{2V_{extremal}}{3M_*^3}e^{2\kappa_5\phi_0}|y|\Big)\,, \end{aligned} \tag{13}$$

where ϕ_0 is an integration constant. We have checked explicitly that the solutions with $4D$ Poincare symmetry are the only allowed solutions which are $4D$ maximally symmetric. Making a more general ansatz for the metric with the symmetries of $4D$ (Anti-) de-Sitter space with curvature h, it is straightforward to show that if the scalar field coupling to the brane is $\exp(\zeta\kappa_5\phi)$, the $4D$ curvature is

$$h^2 = a'^2(0) - \frac{\kappa_5^2\tilde{\phi}'^2(0)}{4}a^2(0)\,, \tag{14}$$

which vanishes by virtue of our choice of the conformal coupling coefficient $\zeta = 2$ in eq. (2). This coupling *prohibits* both de-Sitter symmetry and Anti-de-Sitter symmetry on the brane, regardless of the details of the SM physics. The solutions (13) give a flat Minkowski space, for any value of $V_{extremal}$ even after all quantum corrections are included. Further, altering the parameters of SM, such as for example the electron mass and the fine structure constant, will not break the Poincare symmetry.

The fact that we have found a solution with $4D$ Poincare invariance, robust against SM loops and couplings, does not complete our task. For instance, we can always find $4D$ flat solutions for a 3-brane carrying a range of tensions in $6D$, the tension just inducing a deficit angle in the bulk. The problem is that gravity remains six-dimensional at long distances. Therefore, we must ask whether our $5D$ set-up leads to $4D$ gravity at long distances. If it does, then our Poincare invariant solution demonstrates that the effective $4D$ gravitational dynamics has vanishing cosmological constant.

An important feature of our solution which gives hope for ensuring macroscopic $4D$ gravity is the appearance of a naked curvature singularity at finite proper distance,

$$y_s = \frac{3}{2M_*}e^{-2\kappa_5\tilde{\phi}_0} = \frac{3M_*^3}{2V_{extremal}}e^{-2\kappa_5\phi_0}, \tag{15}$$

away from the brane. Similar singularities have appeared in the work of [16, 17]. The appearance of our singularity can be understood by noting the remarkable analogy [13]

between our equations and Einstein's equations in cosmological Friedmann-Robertson-Walker (FRW) spacetimes (with four spatial dimensions), if we interpret the coordinate y as a cosmological "time" and the warp factor as the FRW "scale factor". The bulk equations (11) then coincide with the cosmological equations of the FRW universe dominated by a massless scalar field. This should not be a surprise, since the ansatz of (10) is a Wick rotation of the FRW ansatz. The analogy with cosmological dynamics immediately shows that away from the brane only two possibilities can occur. One possibility is that the warp factor a monotonically increases forever and the scalar field dissipates away, which means that the topology of the extra dimension is not compact and manifestly has infinite volume. This case however requires negative energy on the brane. Our solution follows from $V_{extremal} > 0$, namely the warp factor monotonically decreases to zero at a finite distance from the brane, where the curvature and the scalar field diverge. This FRW analogy can be thought of in two ways related by time reversal: either the brane specifies initial conditions which evolve into a "Big Crunch", or a "Big-Bang" evolved with final conditions specified by the brane.

Clearly our solution cannot be trusted for $y > y_s$. A careful discussion of the singularity becomes crucial in deciding whether gravity becomes four-dimensional at distances larger than y_s. That we need to do this at all is already striking: the singularity of our solution forces the *short-distance* properties of quantum gravity to become relevant to whether we recover the inverse square law for gravity *at long distances*! We will assume that the singularity is smoothed out by the true short-distance theory of gravity. Since we do not know the details of this theory, we will not be able to make any rigorous claims about whether the resolution of the singularity does what we want. Nevertheless, we see that even without a detailed understanding of the physics which smooths out the singularity, there are two qualitatively different possibilities. The first is an analog of a "Big Crunch/Big Bang" transition. That is, the theory extends across the singularity into a region where a weakly coupled Einstein gravity description is valid again, and the warp factor $a(y)$ diverges as $y \to \infty$. In this case the singularity *does not* end space, and clearly there is no $4D$ gravity at long distances. The second and more attractive possibility is that space ends with a finite volume at the singularity (the analog of time starting at the Big Bang). For instance, short-distance effects might smoothly join the region of high curvature to a highly curved AdS_5 geometry; pushing the singularity to infinite proper distance while preserving a finite volume space. In this case, the extra dimension is effectively compactified in the manner of [6], and we should get $4D$ gravity at long distances. Alternatively, any form of spacetime description might become impossible beyond y_s. In either of these cases, the zero-mode tensor fluctuations

$$ds_5^2 \;=\; (1 - \frac{2M_*}{3} e^{2\kappa_5 \tilde{\phi}_0}|y|)^{1/2} \overline{g}_{\mu\nu}(x) dx^\mu dx^\nu + dy^2, \tag{16}$$

would correspond to a massless $4D$ graviton, with a *finite* $4D$ Planck scale,

$$M_{Pl}^2 = M_*^3 \int_0^{y_s} a^2(y) = M_*^2 e^{-2\kappa_5 \tilde{\phi}_0} = \frac{M_*^6}{V_{extremal}} e^{-2\kappa_5 \phi_0}\,. \tag{17}$$

Similarly the integration constant $\tilde{\phi}_0$ is promoted to a $4D$ scalar zero-mode $\tilde{\phi}_0(x)$[1].

Summarizing: we have found a solution with a 3-brane in $5D$, interacting with bulk gravity and a massless scalar. The bulk couplings are carefully chosen to guarantee $4D$ flat space solutions, while prohibiting other maximally symmetric solutions with $4D$ (Anti-) de Sitter symmetry. The solutions have a curvature singularity at a finite distance from the brane, analogous to the Big Bang or Big Crunch singularity of FRW cosmology. Just as it is possible to think of the crunch singularity as "ending time", it is natural to assume that our singularity ends space, effectively compactifying the extra dimension and giving $4D$ gravity at long distances. *If this assumption is correct*, our model represents a partial solution to the Cosmological Constant Problem, where SM fine-tuning is avoided. Note that as the value of the SM vacuum energy, the warp factor adjusts itself to maintain the vanishing of the $4D$ cosmological constant. This is similar in spirit to the ideas of [12, 13, 14]. Unlike the proposal of [12, 13, 14], in our model the SM fields reside on the "Planck" brane, rather than in the bulk.

But how can we be sure that the same short-distance physics which resolves the singularity does not upset our mechanism ensuring the appearance of a Poincare invariant solution, and hence vanishing $4D$ cosmological constant, without SM fine-tuning? A way of "resolving" the singularity which does not work is by simply ending space with a second brane. For instance, a standard orbifold compactification with a second brane cutting off the singular region with negative energy V_{neg} does require the precise fine-tuning, $-V_{neg} = V_{extremal}$ [19, 5]. Alternatively one can note that our bulk Lagrangian, with the special scalar couplings, arises precisely from compactifying one of the dimensions of a $6D$ theory, with the 3-brane coupling only to the induced metric, and ϕ being the modulus of the extra dimension. It is straightforward to lift our $5D$ solution into the full $6D$ theory. The $6D$ space is compact and locally flat away from our brane, except at a conical singularity corresponding to our singularity at y_s. If this conical singularity is described from the outset as a brane in the theory, its tension must again be fine-tuned against the SM vacuum energy.

We will formulate the conditions for the short distance physics under which our mechanism works. First, we assume that in the absence of the SM 3-brane the full bulk dynamics admits $4D$ Poincare invariant solutions describing $5D$ spacetime ending in the singularity region. We will focus on these because they are the only ones compatible with the matching conditions on the brane. In the analog FRW picture, this amounts to saying that any spatially flat universe can end in a Big Crunch [13]. We also assume that the values of ϕ and ϕ' emerging from the brane are always compatible with the physics of the singularity. In the FRW analogy, this corresponds to saying that a few Planck times after the Big Bang, a scalar field without a potential can emerge with some $\dot{\phi}$, but with any ϕ. The resolutions of the singularity discussed above failed because they violated these requirements. A sufficient but not necessary condition for the second assumption to be

[1] Of course, a massless, gravitationally coupled scalar field is experimentally excluded. In a realistic theory, this scalar will have to pick up a mass of at least $\sim(\mathrm{mm})^{-1}$.

valid is if the short distance physics resolving the singularity is shift symmetric under $\phi \to \phi+$ constant. Also note that the contribution to the $4D$ cosmological constant from higher derivative bulk operators is Planck-suppressed relative to $V_{extremal}$ and therefore of the order we are already neglecting in this paper, as discussed in Eq. (1).

A physical picture of our mechanism follows from the observation that shift symmetry of the bulk action implies an associated conserved current,

$$J^M = -\frac{1}{\sqrt{g_5}} \frac{\delta S_{bulk}}{\delta \partial_M \phi}. \tag{18}$$

$4D$ Poincare invariance implies that only J_y can be non-zero. The fact that the ϕ-couplings to the SM on the brane explicitly break shift symmetry corresponds to a brane-localized source for J, thereby determining the current to be

$$J_y = 3\phi' = M_*^{5/2} e^{2\kappa_5 \tilde{\phi}} = \frac{V_{extremal}}{M_*^{3/2}} e^{2\kappa_5 \phi}. \tag{19}$$

Thus we see that in our set-up the SM vacuum energy is converted into a current emerging from the 3-brane and ending in the singularity region. While the vacuum energy does not show up as $4D$ curvature, the gravitational backreaction of the associated current warps $5D$ spacetime and gives rise to the singularity into which it pours. As discussed, the singularity might be resolved into a highly curved AdS_5 geometry so that the current has room to continue infinitely far away from the brane, or possibly spacetime really ends at the singularity, in which case there may be some non-perturbative breakdown of shift symmetry which allows the current to end.

We conclude by making some comments on the long-distance $4D$ effective theory in our set-up. While we do not know the details of short-distance gravity and are forced to speculate on the nature of the singularity, the physics must yield a consistent $4D$ effective field theory. In particular, this theory must reflect the property of our mechanism that all the extrema of $V(H)$ lead to $4D$ flat solutions. Therefore, the naive guess that the effective theory simply contains the term

$$\int d^4x \sqrt{g_4} \Big(V_{eff}(H) - V_{extremal} \Big) \tag{20}$$

cannot work because of the possibility of multiple extrema $V(H_1), V(H_2), \ldots$. If we subtract the extremum from one vacuum, any other extremum would gravitate. If H is the only light field in the theory, the Lagrangian (20) is the only one consistent with the flat space limit. However, the presence of other light gravitationally coupled fields ψ offers a way out. For example, the true effective $4D$ potential may have the form $V_{eff}(H, \psi) = F(\psi) V_4(H) - G(\psi)$. In the limit where $M_{Pl} \to \infty$, the ψ decouple and the H dynamics is governed by $V_{eff}(H)$. However, with finite M_{Pl}, it is possible for $V_4(H, \psi)$ to only have extrema at points $(H_1, \psi_1), (H_2, \psi_2)$ with vanishing potential, while (H_2, ψ_1), (H_1, ψ_2) are not extrema because they do not satisfy the ψ equation of motion.

It is straightforward to choose functions F and G with this property. In our model a natural candidate for the modulus is ϕ; furthermore, such a potential between ϕ and the electroweak symmetry breaking sector can naturally induce a mass for ϕ of order $(\mathrm{TeV})^2/M_{Pl} \sim (\mathrm{mm})^{-1}$ as required phenomenologically.

In this paper, we have presented a model with a 3-brane in five dimensions, whose only maximally symmetric solutions are $4D$ Poincare invariant, independent of the SM parameters. Our solutions are forced into a strong curvature region which connects the fate of long distance gravity with its short distance properties. With specific assumptions on the nature of the singularity, we recover macroscopic $4D$ gravity with vanishing cosmological constant, in a manner consistent with a $4D$ effective field theory. A better understanding of the singularity would allow us to establish or exclude this idea.

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Note added: While we were completing this paper, we were informed of the upcoming work [22] which overlaps with the ideas presented here.

References

[1] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).

[2] V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett.* **B125**, 136 (1983); *Phys. Lett.* **B125**, 139 (1983).

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429**, 263 (1998); *Phys. Rev.* **D59**, 086004 (1999).

[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436**, 257 (1998).

[5] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).

[6] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).

[7] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, *Phys. Rev. Lett.* **84**, 586 (2000);
J. Lykken and L. Randall, hep-th/9908076.

[8] R. Sundrum, *Phys. Rev.* **D59**, 085010 (1999);
N. Arkani-Hamed, S. Dimopoulos, J. March-Russell, hep-th/9809124.

[9] N. Kaloper, *Phys. Rev.* **D60**, 123506 (1999).

[10] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, hep-th/9909134.

[11] C.P. Burgess, R.C. Myers and F. Quevedo, hep-th/9911164.

[12] H. Verlinde, hep-th/9906182.

[13] J. de Boer, E. Verlinde and H. Verlinde, hep-th/9912012;
E. Verlinde and H. Verlinde, hep-th/9912018.

[14] C. Schmidhuber, hep-th/9912156.

[15] R. Sundrum, *Phys. Rev.* **D59**, 085009 (1999).

[16] M. Gell-Mann and B. Zwiebach, *Nucl. Phys.* **B260**, 569 (1985).

[17] A.G. Cohen and D.B. Kaplan, hep-th/9910132.

[18] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *JHEP* **9905**, 026 (1999);
A. Brandhuber and K. Sfetsos, *JHEP* **9910**, 013 (1999).

[19] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Phys. Rev.* **D59**, 086001 (1999).

[20] W.D. Goldberger and M.B. Wise, *Phys. Rev. Lett.* **83**, 4922 (1999).

[21] A.D. Dolgov, *The very early universe; Proc. 1982 Nuffield Workshop at Cambridge*, ed. G.W. Gibbons, S.W. Hawking and S.T.C. Siklos, (Cambridge University);
F. Wilczek, *Phys. Rep.* **104**, 103 (1984);
R. D. Peccei, J. Sola and C. Wetterich, *Phys. Lett.* **B195**, 183 (1987);
S. M. Barr and D. Hochberg, *Phys. Lett.* **B211**, 49 (1988).

[22] S. Kachru, M. Schulz and E. Silverstein, to appear.